Income Distribution and Macroeconomics

Oded Galor and Joseph Zeira
The Galor-Zeira Model

Overlapping-Generations economy

$t = 0, 1, 2, 3, ...$

One good

3 factors:

$K$

Physical capital

$L_s$

Skilled Labor

$L_u$

Unskilled Labor
The Galor-Zeira Model

- Overlapping-Generations economy
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- Overlapping-Generations economy
- \( t = 0, 1, 2, 3, \ldots \)
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- Overlapping-Generations economy
- $t = 0, 1, 2, 3, \ldots$
- One good
The Galor-Zeira Model

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- \( t = 0, 1, 2, 3, \ldots \)
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- 3 factors:
The Galor-Zeira Model

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- 3 factors:
  - \( K \equiv \text{Physical capital} \)
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- \( t = 0, 1, 2, 3, \ldots \)
- One good
- 3 factors:
  - \( K \equiv \text{Physical capital} \)
  - \( L^S \equiv \text{Skilled Labor} \)
The Galor-Zeira Model

- Overlapping-Generations economy
- \( t = 0, 1, 2, 3, \ldots \)

One good

3 factors:
- \( K \equiv \) Physical capital
- \( L^s \equiv \) Skilled Labor
- \( L^u \equiv \) Unskilled Labor
Production

\[ Y_t = Y_{st} + Y_{ut} \]

Production in the skilled-intensive sector:

\[ Y_{st} = F(K_{st}, L_{st}) \]

\[ L_{st} = f(k_t) \]

Production in the unskilled-intensive sector:

\[ Y_{ut} = aL_{ut} \]
Production

Total output produced

\[ Y_t = Y_t^s + Y_t^u \]
Production

Total output produced

\[ Y_t = Y_t^s + Y_t^u \]

Production in the skilled-intensive sector:

\[ Y_t^s = F(K_t, L_t^s) \equiv L_t^s f(k_t); \quad k_t \equiv K_t / L_t^s \]
Total output produced

\[ Y_t = Y_s^t + Y_u^t \]

- Production in the skilled-intensive sector:

\[ Y_s^t = F(K_t, L_s^t) \equiv L_s^t f(k_t); \quad k_t \equiv K_t / L_s^t \]

- Production in the unskilled-intensive sector:

\[ Y_u^t = aL_u^t \]
Inverse Demand for Factors

- Capital:
  \[ r_t = f'(k_t) \equiv r(k_t) \]
Inverse Demand for Factors

- **Capital:**
  \[ r_t = f'(k_t) \equiv r(k_t) \]

- **Skilled labor:**
  \[ w_t^s = f(k_t) - f'(k_t)k_t \equiv w^s(k_t) \]
Inverse Demand for Factors

- **Capital:**
  \[ r_t = f'(k_t) \equiv r(k_t) \]

- **Skilled labor:**
  \[ w^s_t = f(k_t) - f'(k_t)k_t \equiv w^s(k_t) \]

- **Unskilled labor:**
  \[ w^u_t = a \equiv w^u \]
Factor Prices

- Small open economy
Factor Prices

- Small open economy
- World interest $= r$
Factor Prices

- Small open economy
- World interest = $r$

\[
\begin{align*}
\Rightarrow \\
rt &= r \\
k_t &= f'^{-1}(r) \equiv k \\
w^s_t &= w^s(k) \equiv w^s
\end{align*}
\]
Factor Prices

- Small open economy
- World interest = $r$

\[ r_t = r \]
\[ k_t = f'^{-1}(r) \equiv k \]
\[ w_t^s = w^s(k) \equiv w^s \]

\[ (r_t, w_t^s, w_t^u) = (r, w^s, w^u) \quad \forall t \]
Individuals

The Galor-Zeira Model

Continuum of measure 1

Each Individual has 1 parent and 1 child

Identical in:
- Preferences
- Innate abilities

Different in:
- Parental income
- Inv't in HC
Individuals

- Continuum of measure 1
Individuals

- Continuum of measure 1
- Each Individual has 1 parent and 1 child
Individuals

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  - Innate abilities
The Galor-Zeira Model

Individuals

- Continuum of measure 1
- Each Individual has 1 parent and 1 child
- Identical in:
  - Preferences
  - Innate abilities
- Differ in:
  - Parental income \Rightarrow\ Inv’t in HC
Member of Generation $t$: Period of Life

- First period of life (Period $t$): [
  - Invest in HC
  - Work as unskilled
]

- Second period of life (Period $t+1$): [
  - Work as unskilled / no inv't in HC
  - Work as skilled / inv't in HC
]
Member of Generation $t$: Period of Life

- First period of life (Period $t$):
  - [invest in HC] or [work as unskilled]
Member of Generation $t$: Period of Life

- **First period of life (Period $t$):**
  - [invest in HC] or [work as unskilled]

- **Second period of life (Period $t + 1$):**
  - [work as unskilled / no inv’t in HC] or [work as skilled / inv’t in HC]
Member of Generation $t$: Endowment and Preferences
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- Time endowment:
Member of Generation t: Endowment and Preferences

- **Time endowment:**
  - 1 units of time in each period

Galor-Zeira Inequality and Growth
Member of Generation \( t \): Endowment and Preferences

- **Time endowment:**
  - 1 units of time in each period

- **Capital endowment:**
Member of Generation $t$: Endowment and Preferences

- **Time endowment:**
  - 1 units of time in each period

- **Capital endowment:**
  - $b_t \equiv$ capital inherited in $1^{st}$ period
Member of Generation $t$: Endowment and Preferences

- **Time endowment:**
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- **Capital endowment:**
  - $b_t \equiv$ capital inherited in $1^{st}$ period

- **Preferences:**

  $$u^t = \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1} \quad \alpha \in (0, 1)$$
Member of Generation $t$: Endowment and Preferences

- **Time endowment:**
  - 1 units of time in each period

- **Capital endowment:**
  - $b_t \equiv$ capital inherited in 1$^{st}$ period

- **Preferences:**
  \[
  u^t = \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1} \\
  \alpha \in (0, 1)
  \]

  $c_{t+1} \equiv$ consumption
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Individuals

Member of Generation t: Endowment and Preferences

- **Time endowment:**
  - 1 units of time in each period

- **Capital endowment:**
  - $b_t \equiv$ capital inherited in 1\textsuperscript{st} period

- **Preferences:**

\[
  u^t = \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1} \quad \alpha \in (0, 1)
\]

- $c_{t+1} \equiv$ consumption
- $b_{t+1} \equiv$ transfers to offspring
Second period budget constraint:

\[ c_{t+1} + b_{t+1} \leq \omega_{t+1} \]
Member of Generation $t$: Budget Constraint

Second period budget constraint:

$$c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

$c_{t+1} \equiv \text{consumption}$
Member of Generation $t$: Budget Constraint

Second period budget constraint:

$$c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

$c_{t+1} \equiv$ consumption

$b_{t+1} \equiv$ transfers to offspring
Member of Generation $t$: Budget Constraint

Second period budget constraint:

$$c_{t+1} + b_{t+1} \leq \omega_{t+1}$$

$c_{t+1} \equiv$ consumption  
$b_{t+1} \equiv$ transfers to offspring  
$\omega_{t+1} \equiv$ wealth in period $t + 1$
Member of Generation $t$: Optimization

\[
\{c_{t+1}, b_{t+1}\} = \arg \max \left[ \alpha \ln c_{t+1} + (1 - \alpha) \ln b_{t+1} \right]
\]

s.t. \quad c_{t+1} + b_{t+1} \leq \omega_{t+1}
Member of Generation $t$: Optimization

$$b_{t+1} = (1 - \alpha)\omega_{t+1}$$
Member of Generation $t$: Optimization

\[ b_{t+1} = (1 - \alpha)\omega_{t+1} \]
\[ c_{t+1} = \alpha\omega_{t+1} \]
Member of Generation t: Optimization

\[ b_{t+1} = (1 - \alpha)\omega_{t+1} \]

\[ c_{t+1} = \alpha\omega_{t+1} \]

Indirect Utility:

\[ v^t = \alpha \ln \alpha \omega_{t+1} + (1 - \alpha) \ln (1 - \alpha) \omega_{t+1} \]
Member of Generation $t$: Optimization

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]
\[ c_{t+1} = \alpha \omega_{t+1} \]

Indirect Utility:  
\[ v^t = \alpha \ln \alpha \omega_{t+1} + (1 - \alpha) \ln(1 - \alpha) \omega_{t+1} \]
\[ = [\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)] + \ln \omega_{t+1} \]
Member of Generation $t$: Optimization

$$b_{t+1} = (1 - \alpha)\omega_{t+1}$$

$$c_{t+1} = \alpha\omega_{t+1}$$

Indirect Utility: $\Rightarrow$

$$v^t = \alpha \ln \alpha \omega_{t+1} + (1 - \alpha) \ln (1 - \alpha) \omega_{t+1}$$

$$= [\alpha \ln \alpha + (1 - \alpha) \ln (1 - \alpha)] + \ln \omega_{t+1}$$

$\Rightarrow$ $v^t$ is monotonic increasing in 2nd period wealth, $\omega_{t+1}$
Member of Generation $t$: Optimization

\[ b_{t+1} = (1 - \alpha)\omega_{t+1} \]
\[ c_{t+1} = \alpha\omega_{t+1} \]

Indirect Utility:  \[ v^t = \alpha \ln \alpha \omega_{t+1} + (1 - \alpha) \ln (1 - \alpha) \omega_{t+1} \]
\[ = [\alpha \ln \alpha + (1 - \alpha) \ln (1 - \alpha)] + \ln \omega_{t+1} \]

\[ v^t \text{ is monotonic increasing in 2nd period wealth, } \omega_{t+1} \]
\[ \implies \text{ maximization of } \omega_{t+1}, \text{ is the basis of occupational choices} \]
Fundamental Assumptions

- Imperfect Capital Markets:
  \[ r < i \]  
  \( r \equiv \text{interest rate for lender} \)  
  \( i \equiv \text{interest rate for borrowers (for inv’t in HC)} \)
Fundamental Assumptions

- Imperfect Capital Markets:
  \[ r < i \]  
  \( r \equiv \) interest rate for lender
  \( i \equiv \) interest rate for borrowers (for inv’t in HC)

- Fixed cost of education (Indivisibility of inv’t in HC)
  \[ h > 0 \quad \theta \in [0, 1] \]
Income: Unskilled Individuals

\[ \omega_u t + 1 = (w_u + b_t) \left( 1 + r \right) + w_u = w_u (2 + r) + (1 + r) b_t \]
Income: Unskilled Individuals

\[ \omega^u_{t+1} = \]
Income: Unskilled Individuals

\[ \omega_{t+1}^u = (w)^u \]
Income: Unskilled Individuals

\[ \omega_{t+1}^u = (w^u + b_t) \]
Income: Unskilled Individuals

\[ \omega_{t+1}^u = (w^u + b_t)(1 + r) \]
Income: Unskilled Individuals

$$\omega_{t+1}^u = (w^u + b_t)(1 + r) + w^u$$
Income: Unskilled Individuals

\[
\omega_{t+1}^u = (w^u + b_t)(1 + r) + w^u
\]

\[
= w^u (2 + r)
\]
Income: Unskilled Individuals

\[
\omega_{t+1}^u = (w^u + b_t)(1 + r) + w^u
\]

\[
= w^u(2 + r) + (1 + r)b_t
\]
Income: Skilled Individuals

\[ \omega^s_{t+1} = \]
Income: Skilled Individuals

\[ \omega_{t+1}^s = \begin{cases} \omega^s \\ w^s \end{cases} \]
Income: Skilled Individuals

\[ \omega_{t+1}^s = \begin{cases} 
& w^s - (h - b_t) 
\end{cases} \]
Income: Skilled Individuals

\[ \omega^s_{t+1} = \begin{cases} 
  w^s - (h - b_t)(1 + i) & \text{if } b_t \leq h 
\end{cases} \]
Income: Skilled Individuals

\[ \omega_{t+1}^s = \begin{cases} 
    w^s - (h - b_t)(1 + i) & \text{if} \quad b_t \leq h \\
    w^s & \text{if} \quad b_t > h 
\end{cases} \]
Income: Skilled Individuals

\[
\omega_{t+1}^s = \begin{cases} 
  w^s - (h - b_t)(1 + i) & \text{if } b_t \leq h \\
  w^s + (b_t - h) & \text{if } b_t > h
\end{cases}
\]
Income: Skilled Individuals

\[
\omega^{s}_{t+1} = \begin{cases} 
  w^s - (h - b_t)(1 + i) & \text{if } b_t \leq h \\
  w^s + (b_t - h)(1 + r) & \text{if } b_t \geq h 
\end{cases}
\]
Income: Skilled Individuals

\[
\omega_{t+1}^s = \begin{cases} 
  w^s - (h - b_t)(1 + i) & \text{if } b_t \leq h \\
  w^s + (b_t - h)(1 + r) & \text{if } b_t \geq h 
\end{cases}
\]

\[
\Rightarrow \omega_{t+1}^s = \begin{cases} 
  w^s - (1+i)h + (1+i)b_t & \text{if } b_t \leq h 
\end{cases}
\]
Income: Skilled Individuals

\[ \omega_{t+1}^s = \begin{cases} 
  w^s - (h - b_t)(1 + i) & \text{if } b_t \leq h \\
  w^s + (b_t - h)(1 + r) & \text{if } b_t \geq h 
\end{cases} \]

\[ \implies \]

\[ \omega_{t+1}^s = \begin{cases} 
  w^s - (1 + i)h + (1 + i)b_t & \text{if } b_t \leq h \\
  w^s - (1 + r)h + (1 + r)b_t & \text{if } b_t \geq h 
\end{cases} \]
Assumptions

- Investment in human capital is *not* beneficial for individuals who must finance the entire cost of education via borrowing

$$w^s - (1 + i)h < 0$$  \hspace{1cm} (A3)
Assumptions

- Investment in human capital is *not* beneficial for individuals who must finance the entire cost of education via borrowing
  
  \[ w^s - (1 + i)h < 0 \]  
  
  \[ (A3) \]

- Investment in human capital is beneficial for individuals who can finance the entire cost of education *without* borrowing
  
  \[ w^s - (1 + r)h > w^u(2 + r) \]  
  
  \[ (A4) \]
Income from Being Unskilled Worker

\[ \omega_{i+1}^u = w^u (2 + r) + (1 + r) b_i \]
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Income from Being Unskilled Worker

\[ \omega_{t+1}^u = w^u (2 + r) + (1 + r)b_t \]
Income from Being Skilled Worker: Borrowers

\[ w^s - (1 + i)h < 0 \quad (A3) \]
Income from Being Skilled Worker: Borrowers

\[ \omega_{t+1}^s = w^s - (1+i)h + (1+i)b_t \quad \text{if} \quad b_t \leq h \]
Income from Being Skilled Worker: Borrowers

\[ \omega_{t+1}^{s} = w^{s} - (1 + i)h + (1 + i)b_{t} \text{ if } b_{t} \leq h \]
Income from Being Skilled Worker: Borrowers

\[ w^s - (1 + r)h > w^u(2 + r) \quad (A4) \]
Income from Being Skilled Worker: Lenders

\[ \omega_{t+1}^s = w^s - (1 + r)h + (1 + r)b_t \quad \text{if} \quad b_t \geq h \]
Bequest and Occupational Choice

\[ \omega_{t+1} = \omega^s(b_t) \]

\[ \omega_{t+1} = \omega^u(b_t) \]
Bequest and Occupational Choice

\[
\omega_{t+1}^s = \omega^s(b_t) \\
\omega_{t+1}^u = \omega^u(b_t)
\]
Bequest and Occupational Choice

\[
\begin{align*}
    b_t & \quad \left\{ \begin{array}{l}
        < f \quad \rightarrow \omega_{t+1}^u > \omega_{t+1}^s \quad \text{(individual} \ t \ \text{becomes unskilled)} \\
        > f \quad \rightarrow \omega_{t+1}^u < \omega_{t+1}^s \quad \text{(individual} \ t \ \text{becomes skilled)}
    \end{array} \right.
\end{align*}
\]
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Occupational Choice

Bequest and Occupational Choice

\[ b_t \begin{cases} < f & \rightarrow \omega_{t+1}^u > \omega_{t+1}^s \text{ (individual } t \text{ becomes unskilled)} \\ > f & \rightarrow \omega_{t+1}^u < \omega_{t+1}^s \text{ (individual } t \text{ becomes skilled)} \end{cases} \]

where

\[ f = \frac{w^u(2 + r) - [w^s - (1 + i)h]}{i - r} > 0 \]
$b_{t+1}$
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]

\[ b_{t+1} = \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha)\omega_{t+1} \]

\[ b_{t+1} = \begin{cases} 
(1 - \alpha)[w^u(2 + r) + (1 + r)b_t] & b_t \in [0, f] 
\end{cases} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]

\[ b_{t+1} = \begin{cases} 
(1 - \alpha)[w^u(2 + r) + (1 + r)b_t] & b_t \in [0, f] \\
(1 - \alpha) & \text{otherwise}
\end{cases} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]

\[ b_{t+1} = \begin{cases} 
(1 - \alpha)[w^u(2 + r) + (1 + r)b_t] & b_t \in [0, f] \\
(1 - \alpha)[w^s - (1 + i)h + (1 + i)b_t] & b_t \in [f, h] 
\end{cases} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha) \omega_{t+1} \]

\[ b_{t+1} = \begin{cases} 
(1 - \alpha) [w^u (2 + r) + (1 + r) b_t] & b_t \in [0, f] \\
(1 - \alpha) [w^s - (1 + i) h + (1 + i) b_t] & b_t \in [f, h] \\
(1 - \alpha) & \end{cases} \]
Bequest Dynamics

\[ b_{t+1} = (1 - \alpha)\omega_{t+1} \]

\[ b_{t+1} = \begin{cases} 
(1 - \alpha)[w^u(2 + r) + (1 + r)b_t] & b_t \in [0, f] \\
(1 - \alpha)[w^s - (1 + i)h + (1 + i)b_t] & b_t \in [f, h] \\
(1 - \alpha)[w^s - (1 + r)h + (1 + r)b_t] & b_t \in [h, \infty] 
\end{cases} \]
Bequest Dynamics: Sufficient Conditions for Multiplicity of Steady-State

\[
(1 - \alpha)(1 + r) < 1
\]

\[
(1 - \alpha)(1 + i) > 1
\]
Bequest Dynamics: Sufficient Conditions for Multiplicity of Steady-State

\[(1 - \alpha)(1 + r) < 1\]  \hfill (A5)

\[(1 - \alpha)(1 + i) > 1\]

\[(1 - \alpha)w^s > h\]  \hfill (A6)
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Bequest Dynamics

\[ \phi(b_t) \]

\[ (1 - \alpha)w^s \]

\[ (1 - \alpha)(2 + r) \]

\[ (1 - \alpha)(1 + i) \]

\[ (1 - \alpha)(1 + r) \]

\[ f \]

\[ h \]

\[ b_t \]

\[ b_{t+1} \]
Bequest Dynamics: Multiple Steady-State Equilibrium

\[ b_{t+1} \]

\[ \phi(b_t) \]

\[ b^u \]

\[ g \]

\[ b^s \]
Bequest Dynamics: Stability of High Bequest Equilibrium
Bequest Dynamics: Stability of Steady-State Equilibria

\[ \phi(b_t) \]

Diagram with axes labeled: 
- \( b_{t+1} \) on the y-axis
- \( b_t \) on the x-axis

Key points:
- \( b^u \)
- \( g \)
- \( b^s \)
The Distribution of the Inheritance in Period $t$

$\xi(b_0)$
\[ \zeta_t(b_t) \equiv \text{Distribution of inheritance at time } t \]

\[ L_t = \int_0^\infty \zeta(b_t) db_t \equiv 1 \]
The Distribution of the Inheritance in Period $t$
Income Distribution of the Long Run Decomposition of the Labor Force

$$\lim_{t \to \infty} l_t^u = \int_0^g \xi_t(b_t) db_t \equiv \bar{l}_t^u$$
Income Distribution of the Long Run Decomposition of the Labor Force

\[ \lim_{t \to \infty} l^u_t = \int_0^g \xi_t(b_t) \, db_t \equiv \bar{l}^u \]

\[ \lim_{t \to \infty} l^s_t = \int_g^\infty \xi_t(b_t) \, db_t \equiv \bar{l}^s \]
Income Distribution of the Long Run Decomposition of the Labor Force

\[
\lim_{t \to \infty} l_t^u = \int_0^g \zeta_t(b_t) \, db_t \equiv \bar{l}^u
\]

\[
\lim_{t \to \infty} l_t^s = \int_g^\infty \zeta_t(b_t) \, db_t \equiv \bar{l}^s
\]

where

\[
\frac{\partial \bar{l}^s}{\partial g} < 0
\]
Income Distribution of the Long Run Decomposition of the Labor Force

\[
\lim_{t \to \infty} l_t^u = \int_0^g \zeta_t(b_t) \, db_t \equiv \bar{l}^u
\]

\[
\lim_{t \to \infty} l_t^s = \int_g^{\infty} \zeta_t(b_t) \, db_t \equiv \bar{l}^s
\]

where

\[
\frac{\partial \bar{l}^s}{\partial g} < 0
\]

and

\[
g = \frac{(1 - \alpha)[(1 + i)h - w^s]}{(1 - \alpha)(1 + i) - 1} > 0
\]
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Inequality and Skill Composition

Income Distribution of Skill Composition

\[ b_{t+1} \]

\[ \phi(b_t) \]

\[ \bar{b}^u \]

\[ \bar{b}^s \]

\[ D_t(b_t) \]

\[ g \]

\[ \bar{l}^u \]

\[ \bar{l}^s \]

\[ b_0 \]
Income Distribution of Skill Composition
Inequality and Development: Rich Economies

\[ \phi(b_t) \]

\[ b_{t+1} \]

\[ b_t \]

\[ g \quad \bar{b}_t \]
Rich Economies: Inequality is Harmful for Development

Inequality reduces human capital formation
Rich Economies: Inequality is Harmful for Development

\[
\phi(b_t) = \left(1 + \phi_1\right) b_t
\]
Rich Economies: Inequality is Harmful for Development
Inequality and Development: Poor Economies

\[
\phi(b_t) = \begin{cases} 
1 & \text{if } b_t < b^* \\
\frac{b_t}{b^*} & \text{if } b^* \leq b_t \leq b^* \\
1 & \text{if } b_t > b^* 
\end{cases}
\]
Poor Economies: Inequality may Benefit Development

Inequality stimulates human capital formation

\[ \phi(b_t) \]

\[ \bar{b}_t \quad g \]
Poor Economies: Inequality may Benefit Development