The Demographic Transition

Oded Galor

September 20, 2019

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- The Mystery of the Gaps
  - What is the origin of the vast inequality in income per capita across countries and regions?

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  - The roots of the differential timing of the transition across the globe
  - The origins of the differential timing of the demographic transition
  - The role of evolutionary forces in the onset of these transitions

Evidence

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- The demographic transition is critical for the understanding of:
  - The timing of the transition from stagnation to growth
  - The vast inequality across countries and regions
- The forces that triggered the onset of the demographic transition
  - Central to the resolution of the mysteries of growth & gaps

Phases of Development

- The Malthusian Epoch

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- The Post-Malthusian Regime

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Phases of Development: Timeline of the Most Developed Economies

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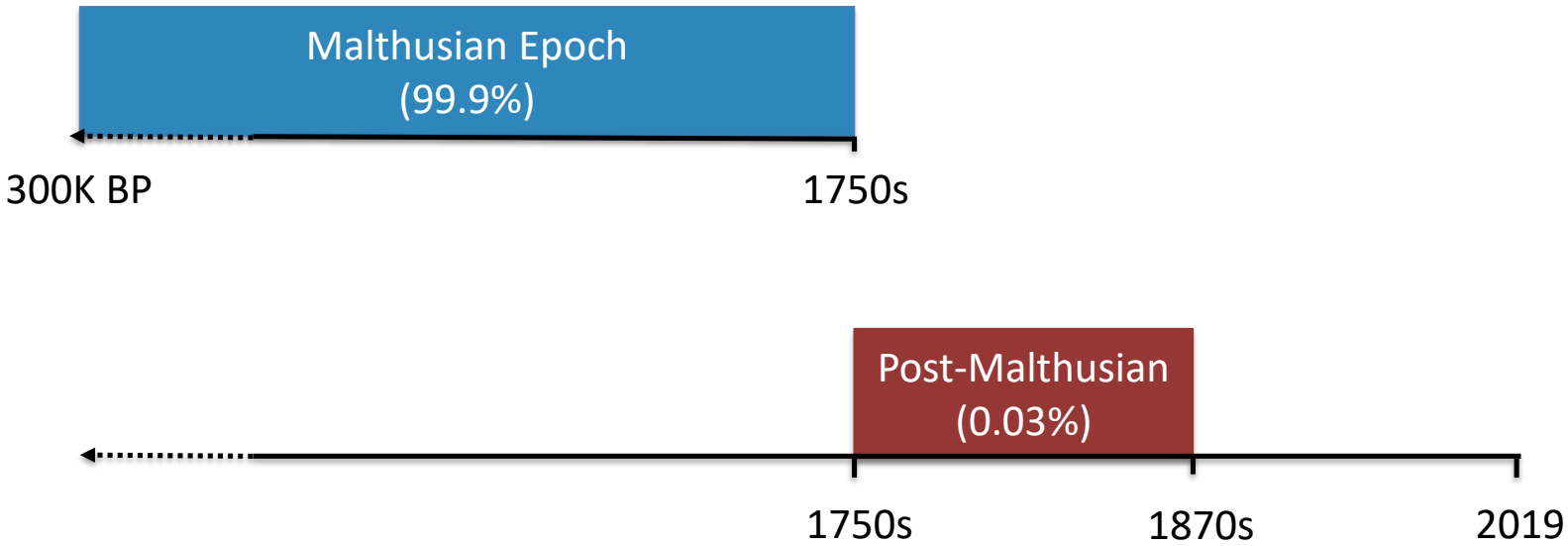

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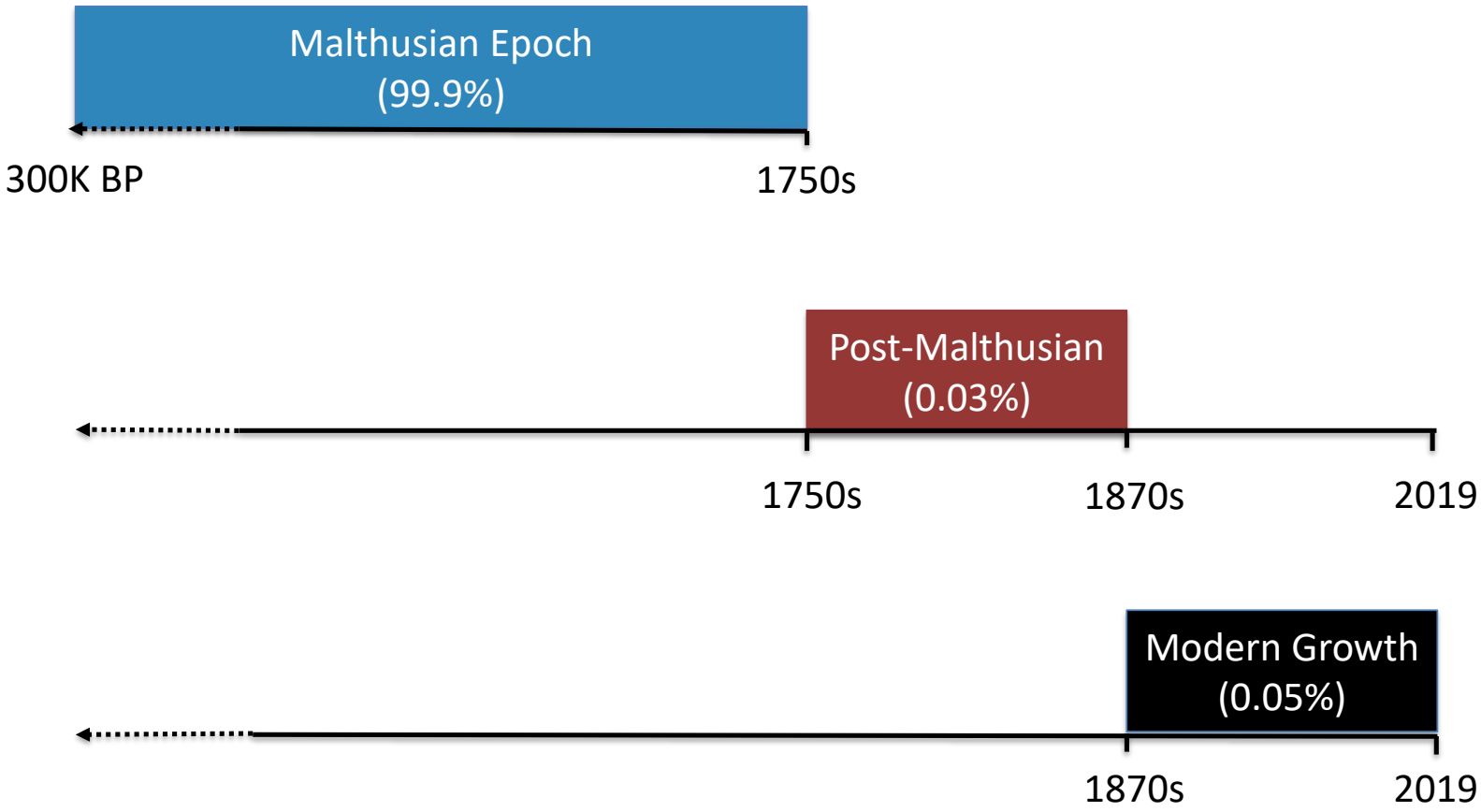

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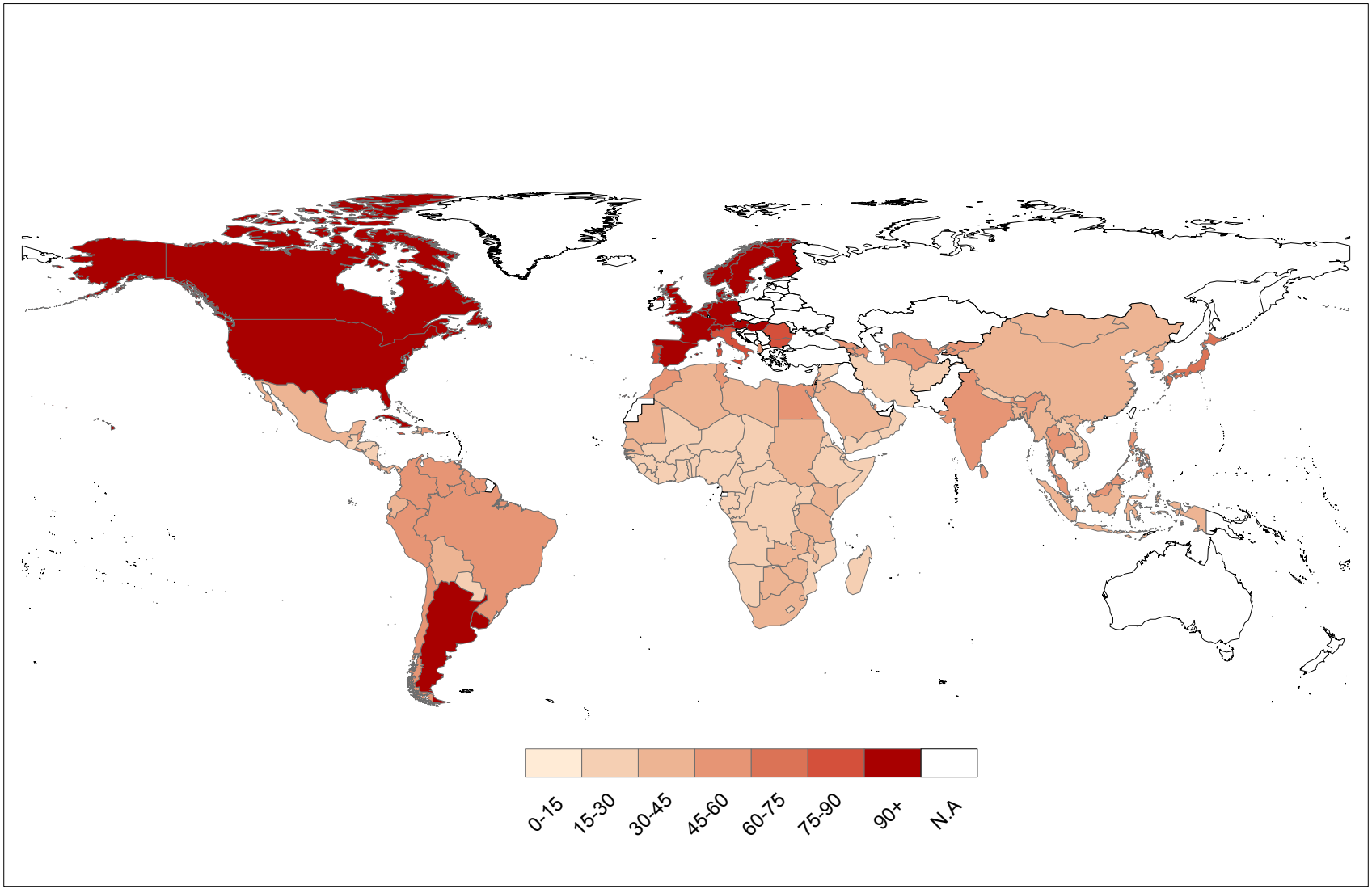
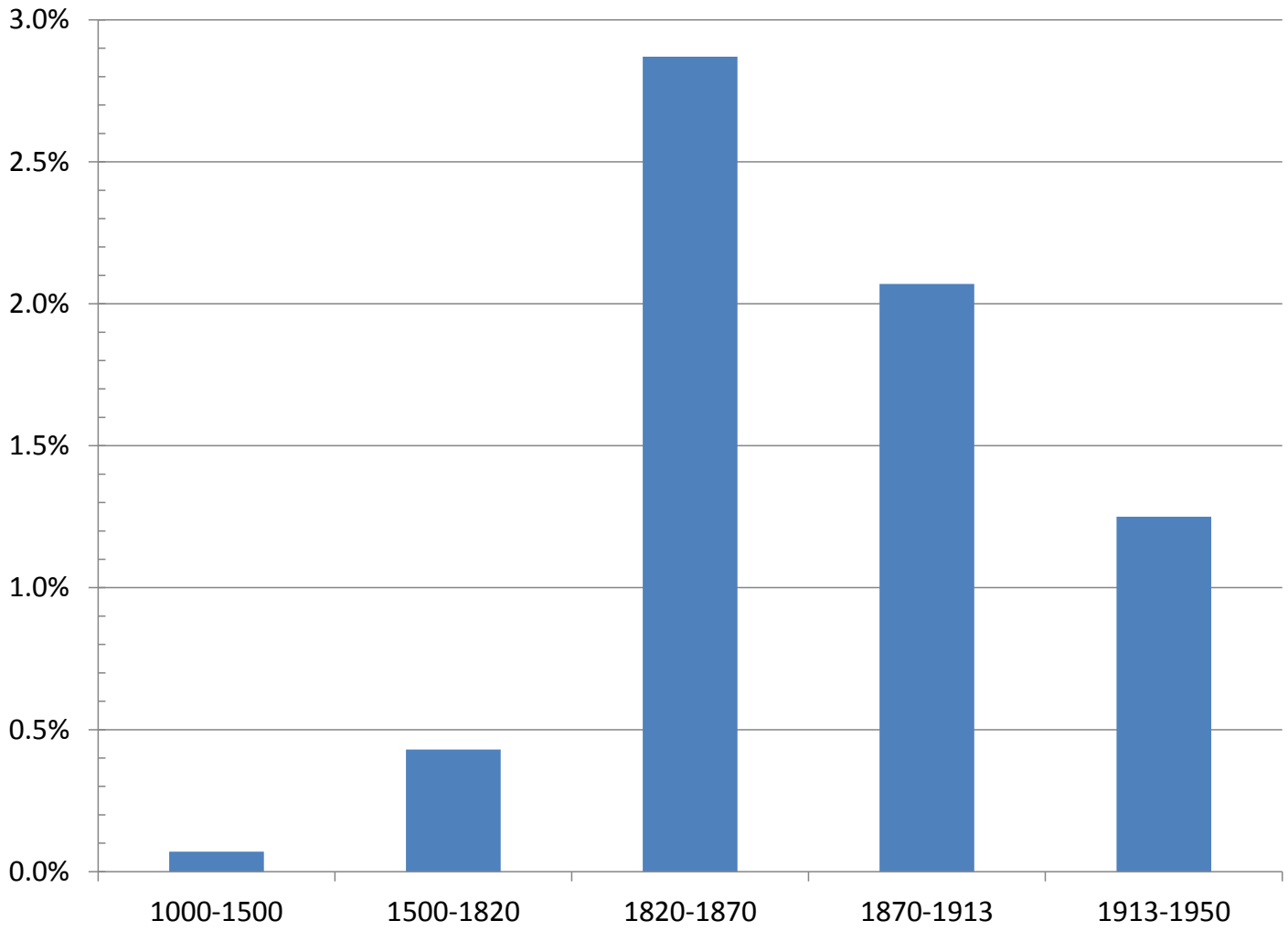
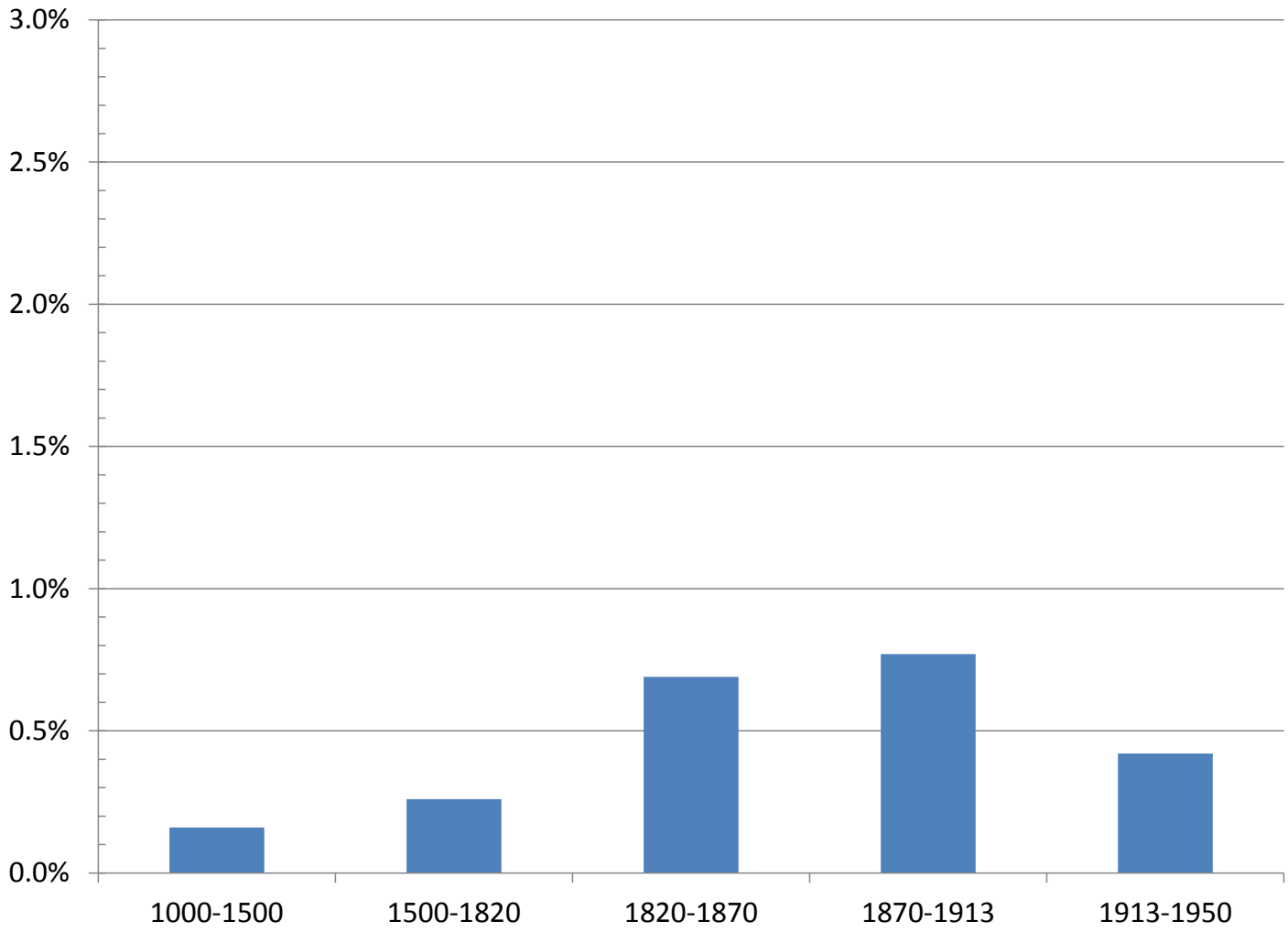
Years Elapsed since the Onset of the Fertility Decline



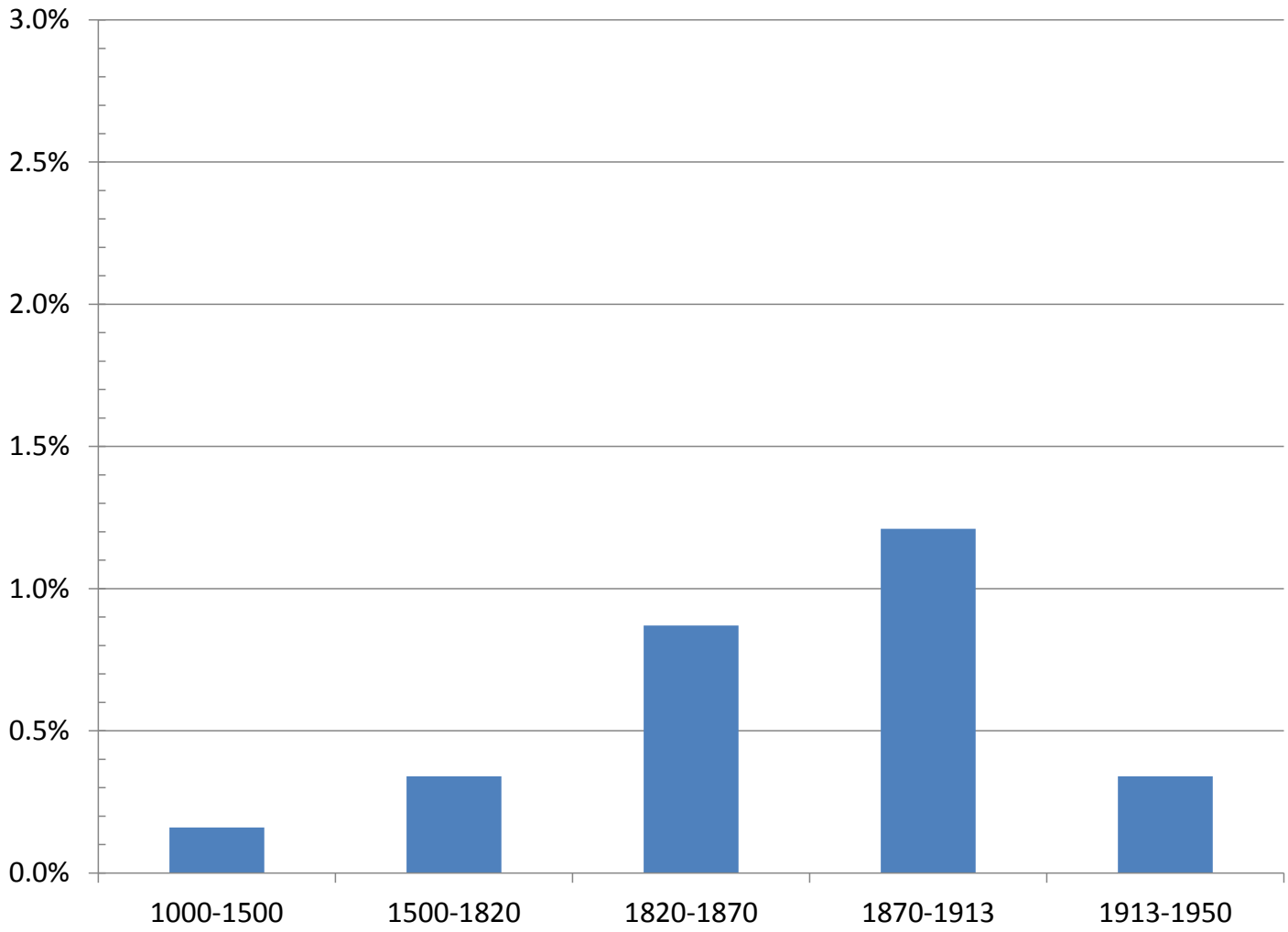
Early Fertility Decline – Western Offshoots



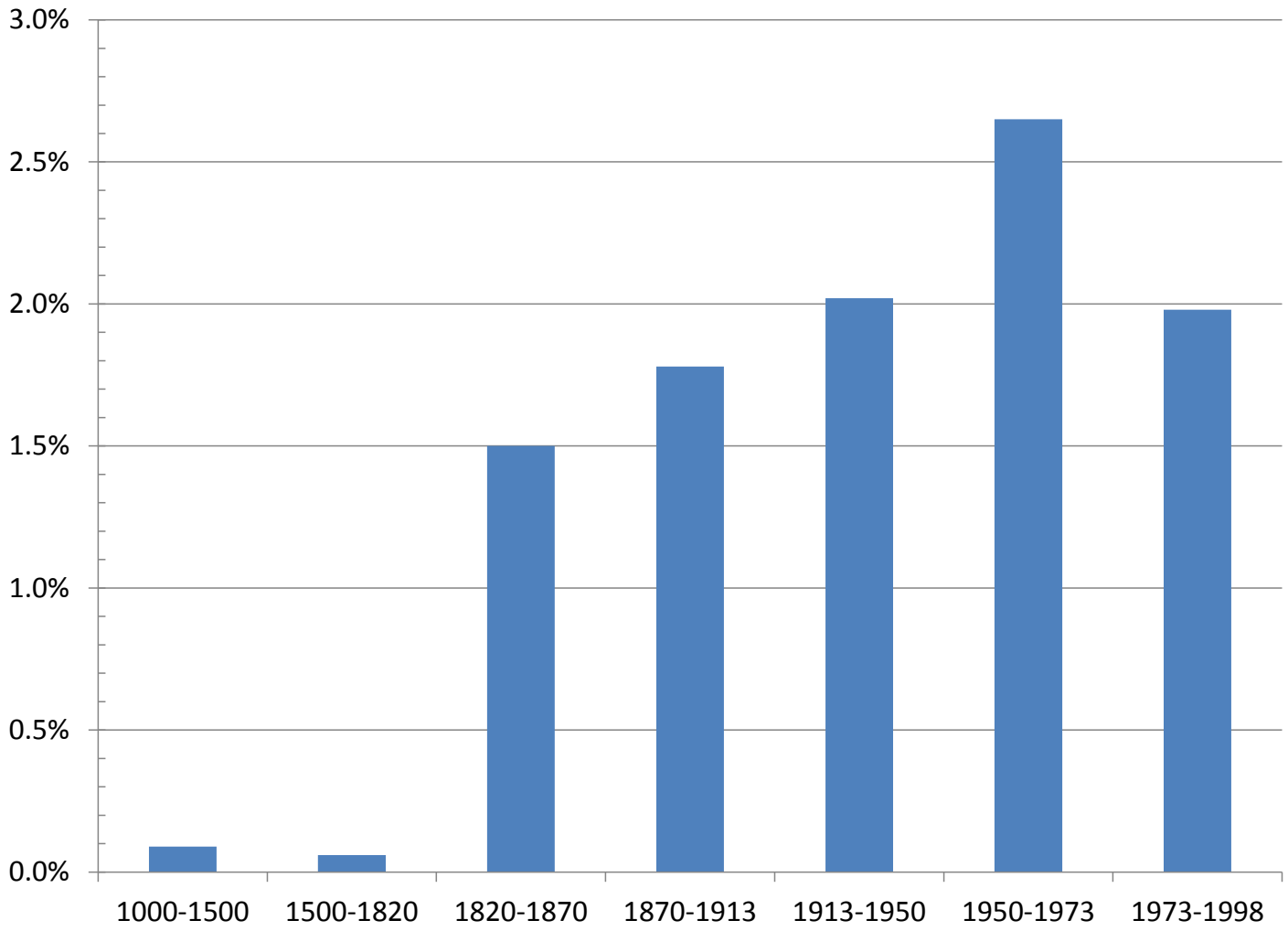
Early Fertility Decline – Western Europe



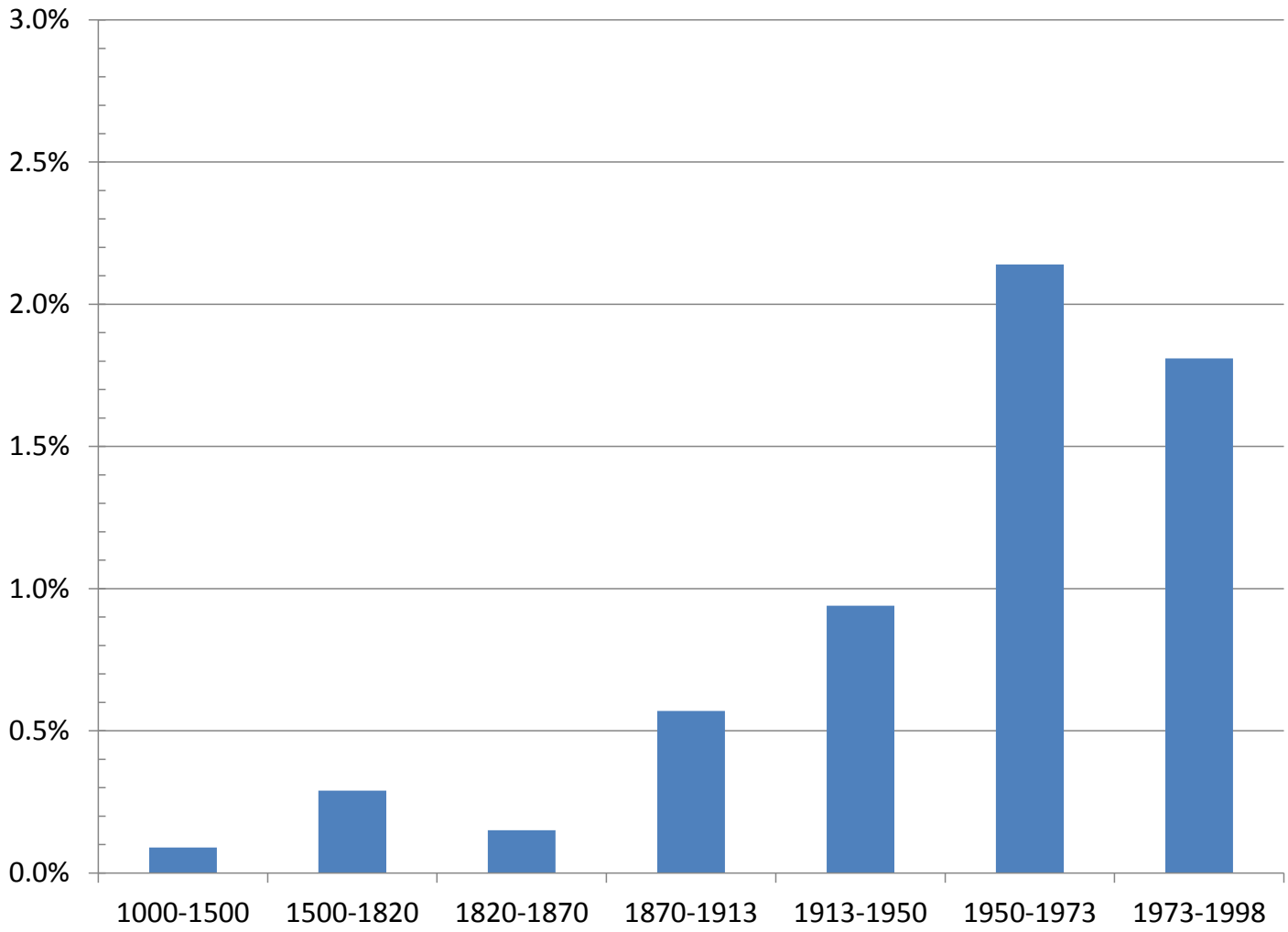
Early Fertility Decline – Eastern Europe



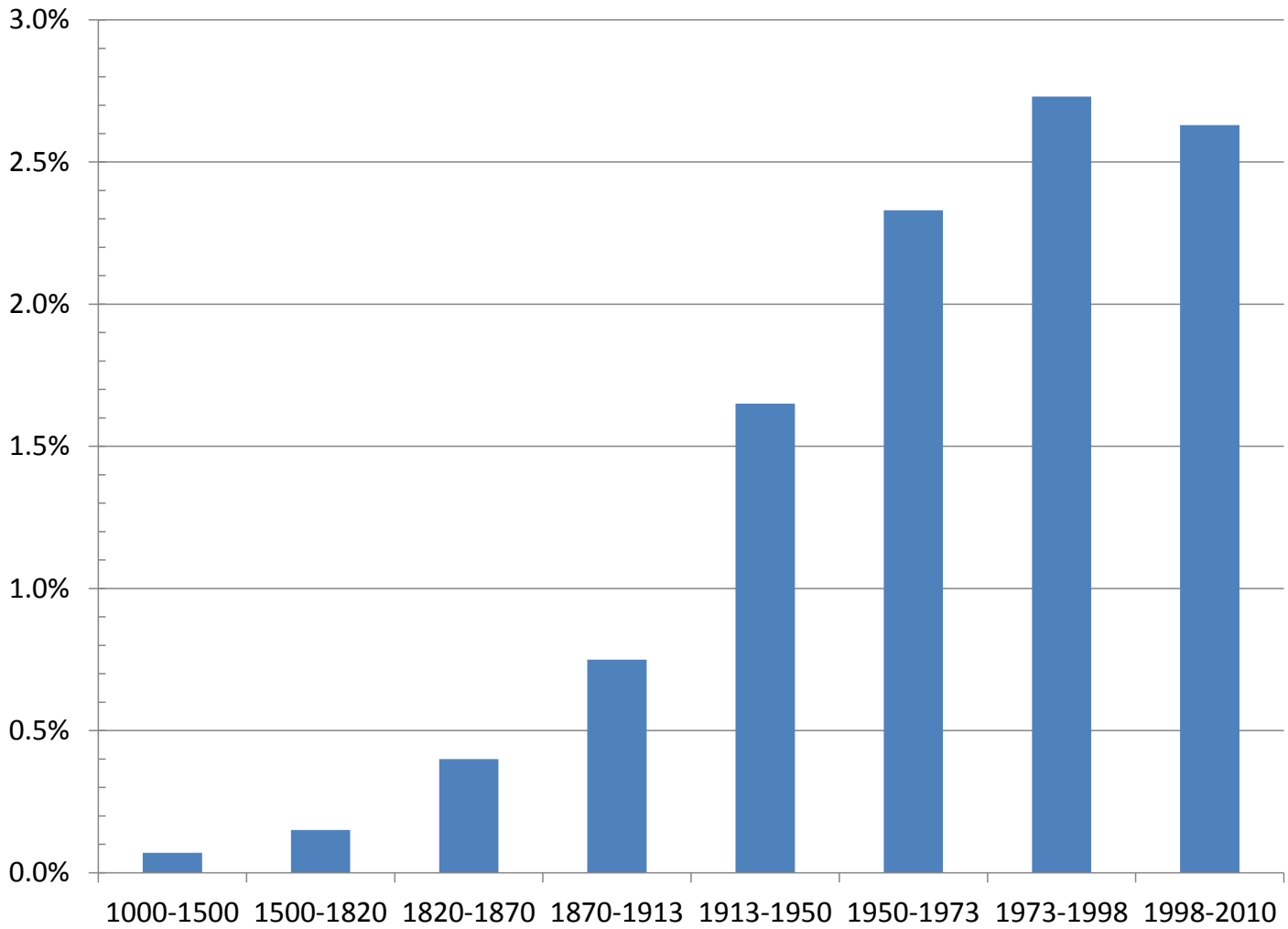
Late Fertility Decline – Latin America



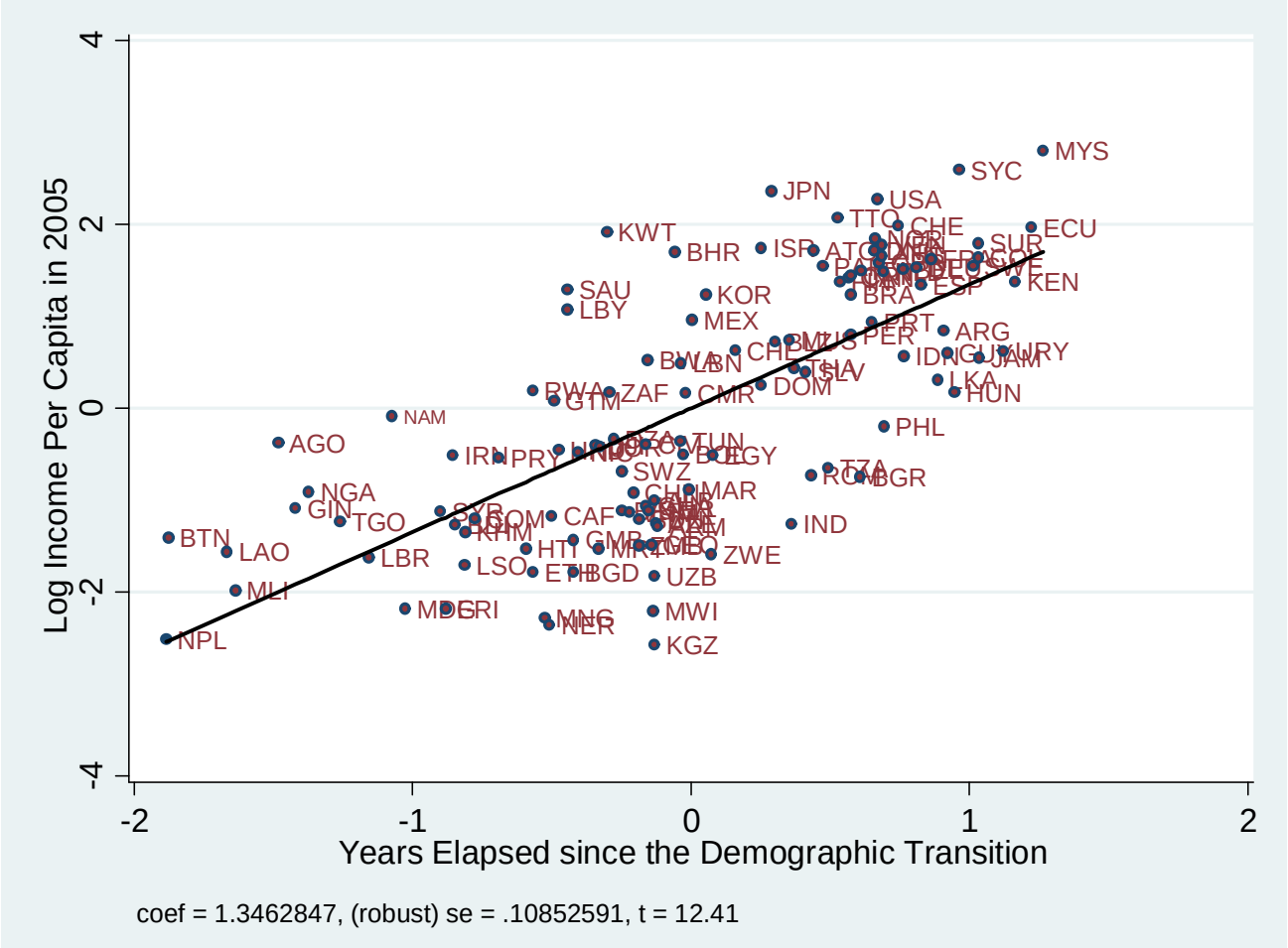
Late Fertility Decline – Asia



Late Fertility Decline – Africa

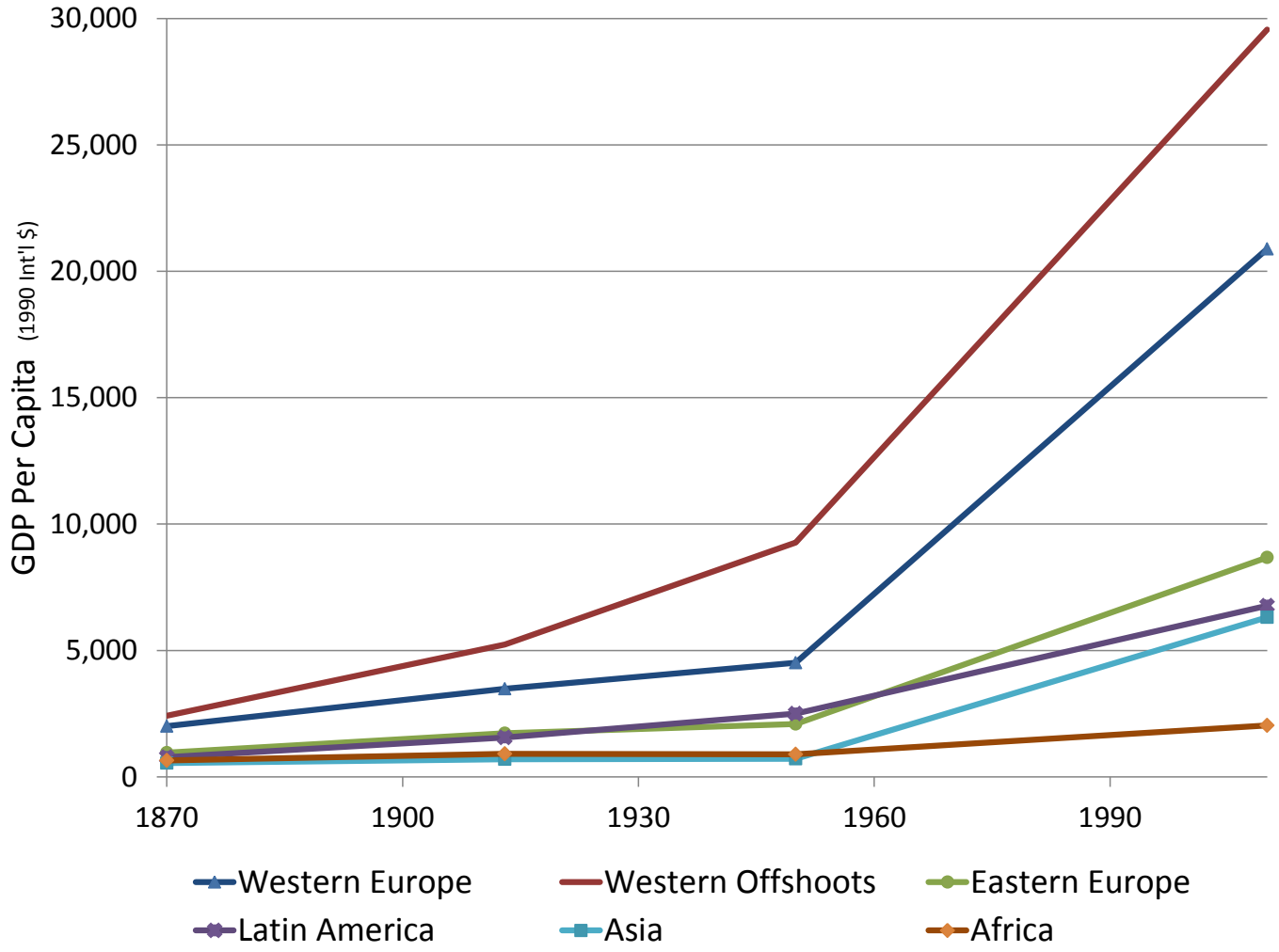


Timing of the Demographic Transition and Current Income per Capita

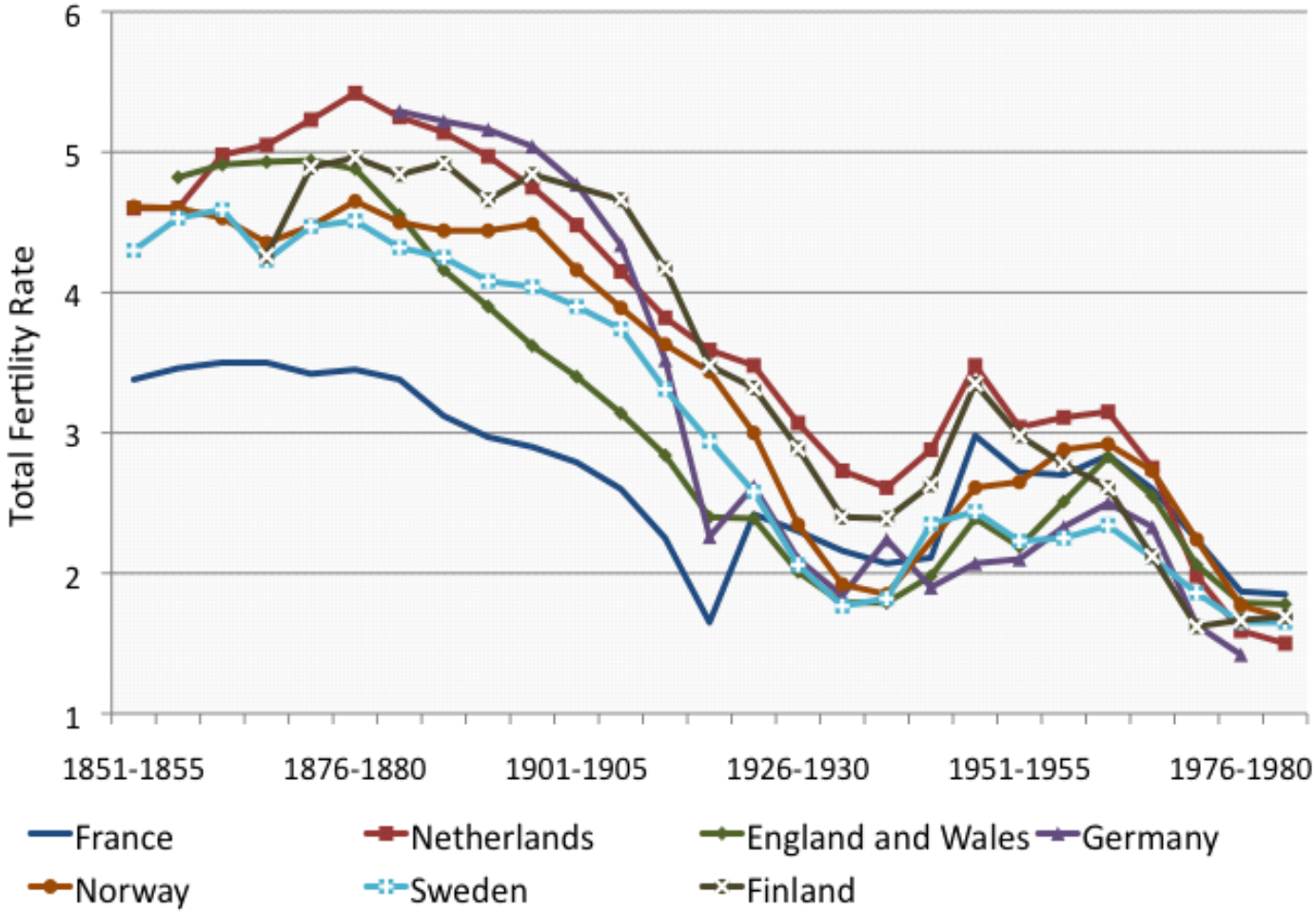


Conditional on absolute latitude.

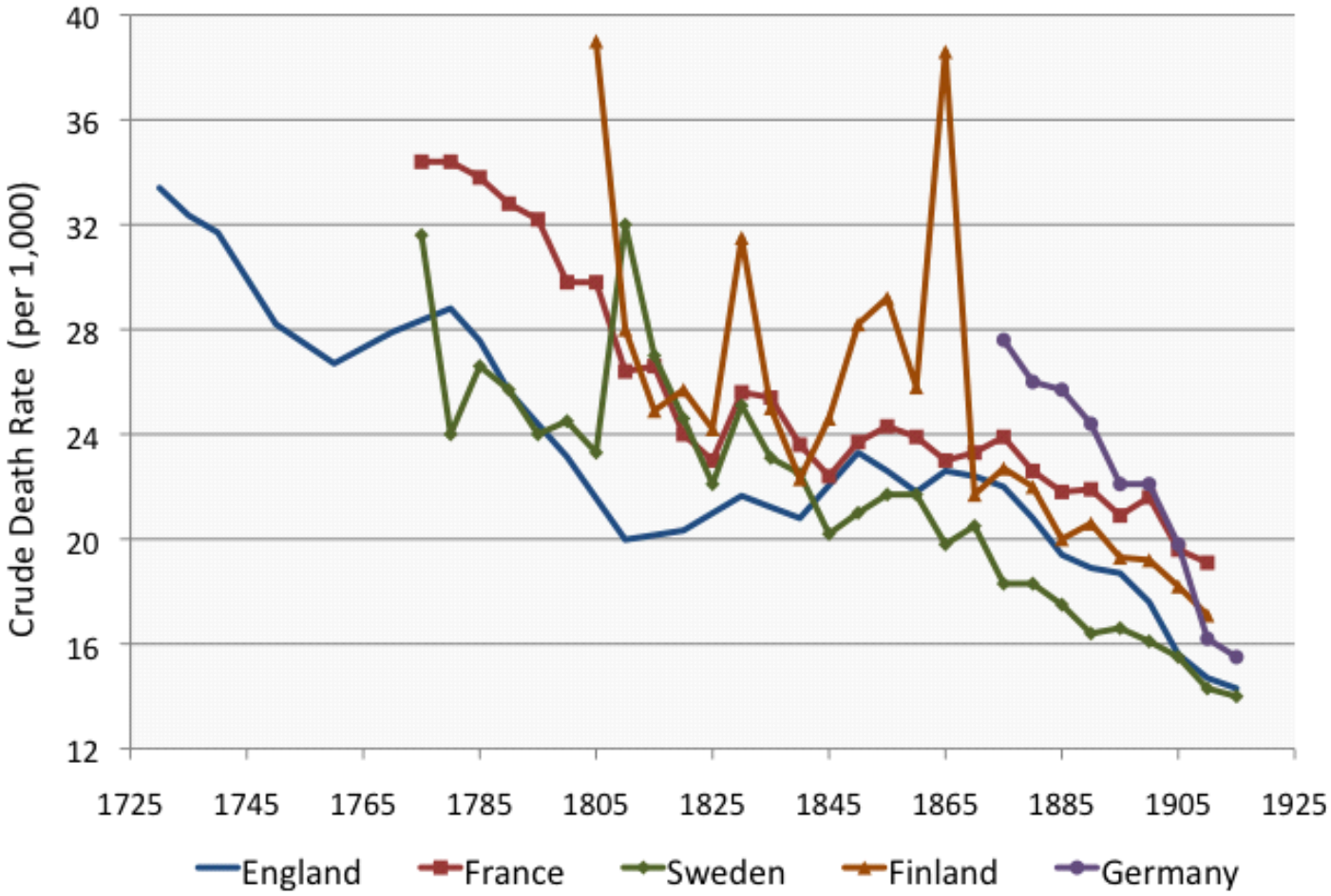
Timing of the Demographic Transition and Divergence across Regions



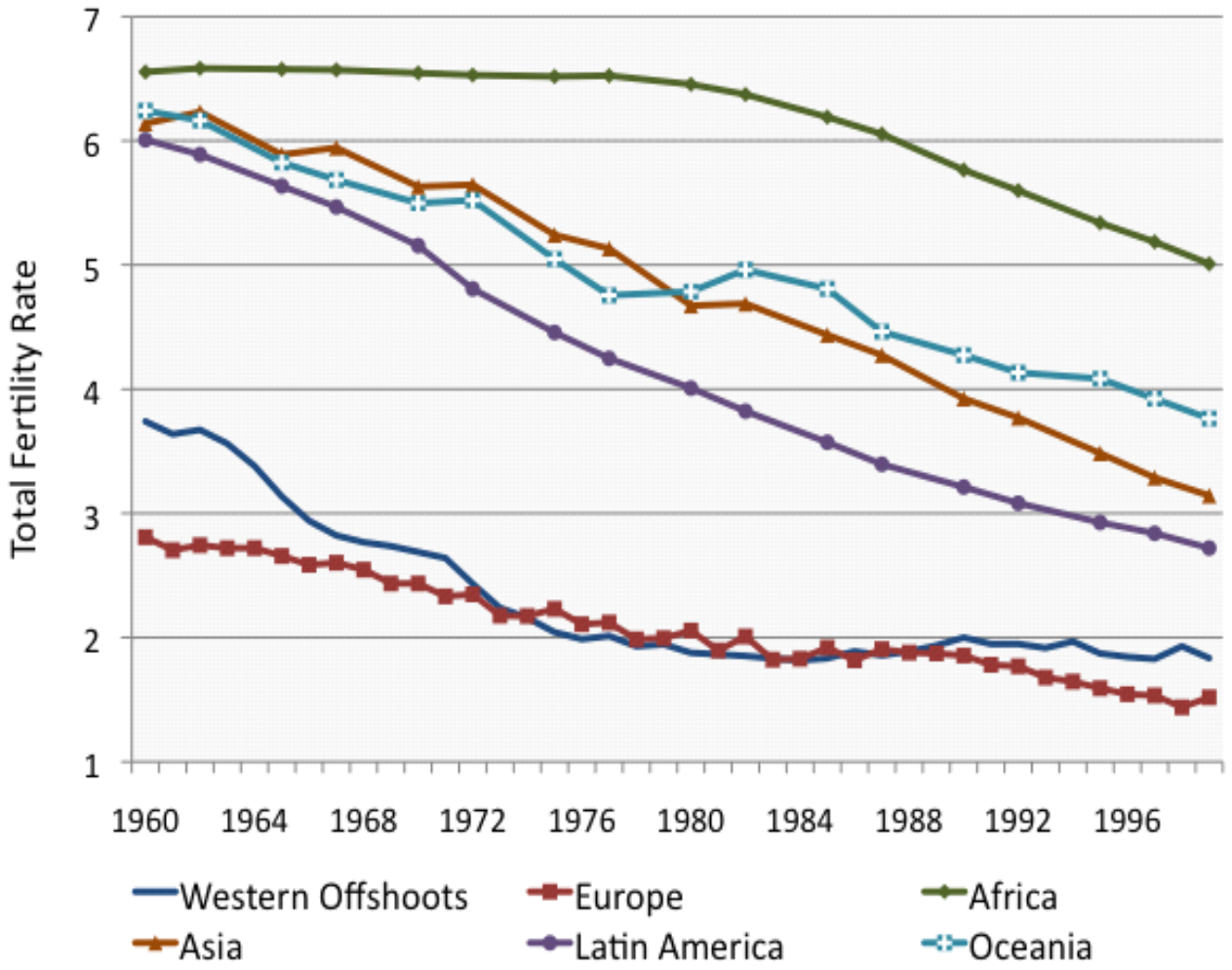
The Demographic Transition in Western Europe: Total Fertility Rates



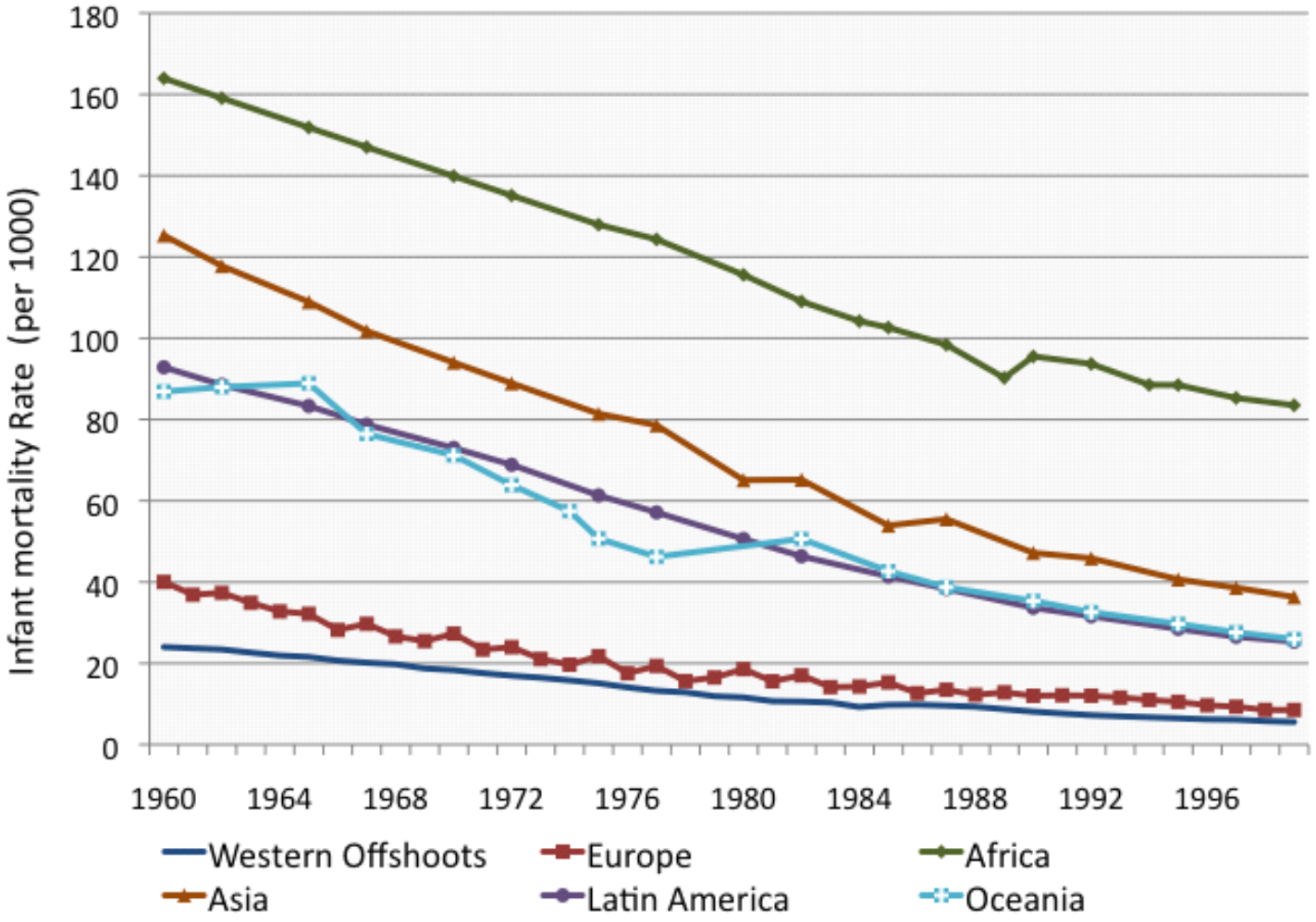
Mortality Decline Western Europe: 1730-1920



The Evolution of Total Fertility Rate across Regions, 1960-1999



Decline in infant mortality rates across regions, 1960-1999



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    - More income can be devoted to raising children
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  - A substitution effect:

$$\Uparrow [y\tau] n + c \leq y$$

    - The opportunity cost of raising children increases
    - operates towards $n \Downarrow$

The Rise in Income: Mechanism

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  - Fertility declines in the process of development

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  - Non-robust (e.g., under homothetic preferences a rise in income will not trigger a fertility decline)

The Rise in Income - Homothetic Preferences

- Preferences:

$$u = n^{\gamma} c^{(1-\gamma)} \qquad 0 < \gamma < 1$$

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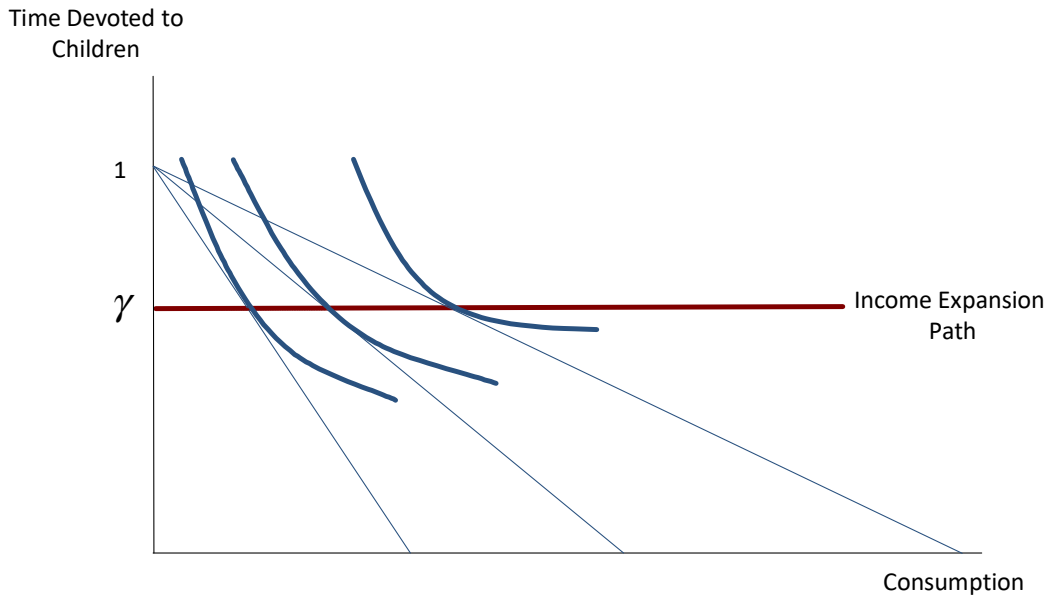
The Rise in Income - Homothetic Preferences



- $1 =$ the household time endowment
- $\gamma =$ The optimal time deveoted to children ($\gamma/\tau =$ optimal nubmer of children)
  - $\Rightarrow$ nubmer of children is indepdendet of the level of income

The Rise in Income: Testable predictions

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  - The timing of the fertility decline is inversely related to the level of income per capita, across countries that are similar in sociocultural characteristics (e.g., countries that sahre simialr non-economic factors that may affect fertility decisions)

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- Within an economy
  - The number of (surviving) children is inversely related to their levels of income across households

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- Western Europe
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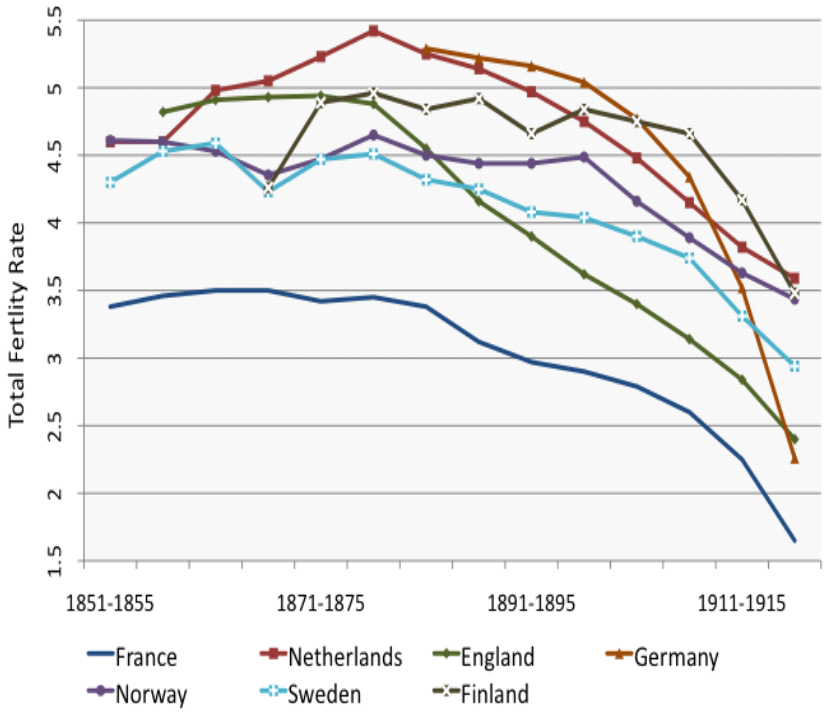
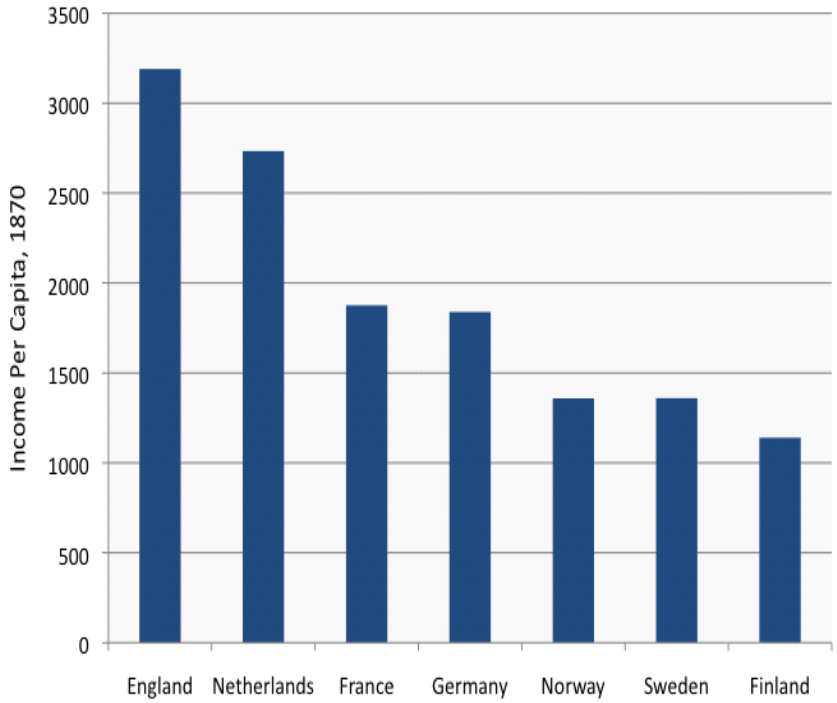
Simultaneous DT desptie Large Differences in Income per Capita: Western Europe 1870s





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- England (1630s)
  - Reproductive success increases with income (Clark (JEH 2006, De la Croix et al.„ JEG 2019)

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  - $\theta \equiv$ child mortality rate

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- Optimal nubmer of children born (TFR - Total Fertility Rate)

$$n^b = \frac{n}{(1-\theta)} = \frac{\gamma}{(1-\theta)\tau}$$

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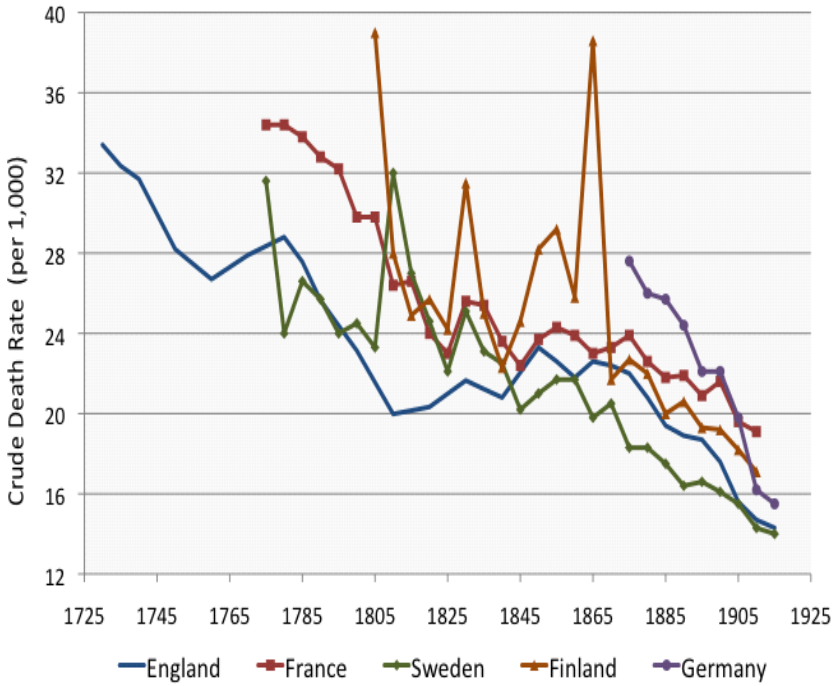
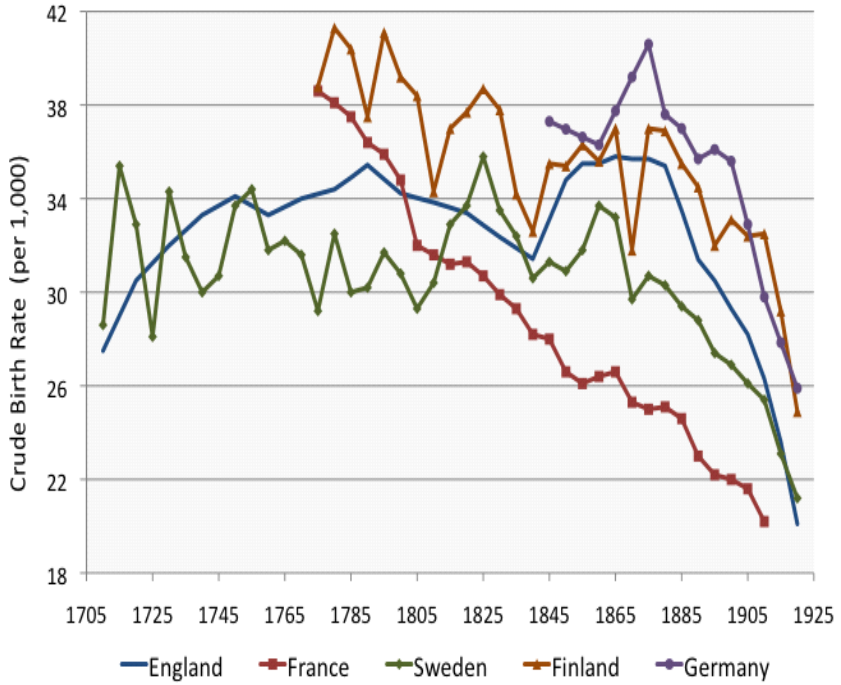
  - No effect on NRR
    - $n = \gamma/\tau$ is independent of θ

The Decline in Mortality and Fertlity (TFR) - Evidence





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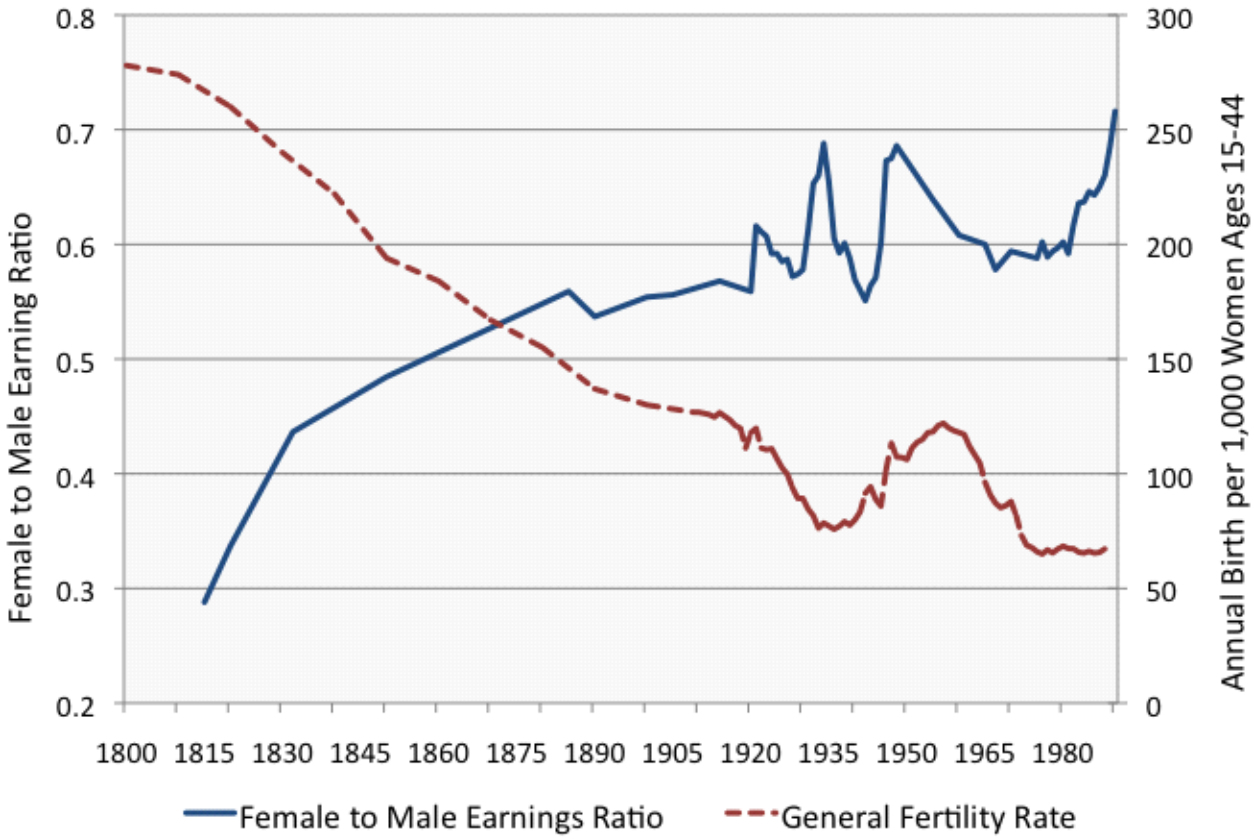
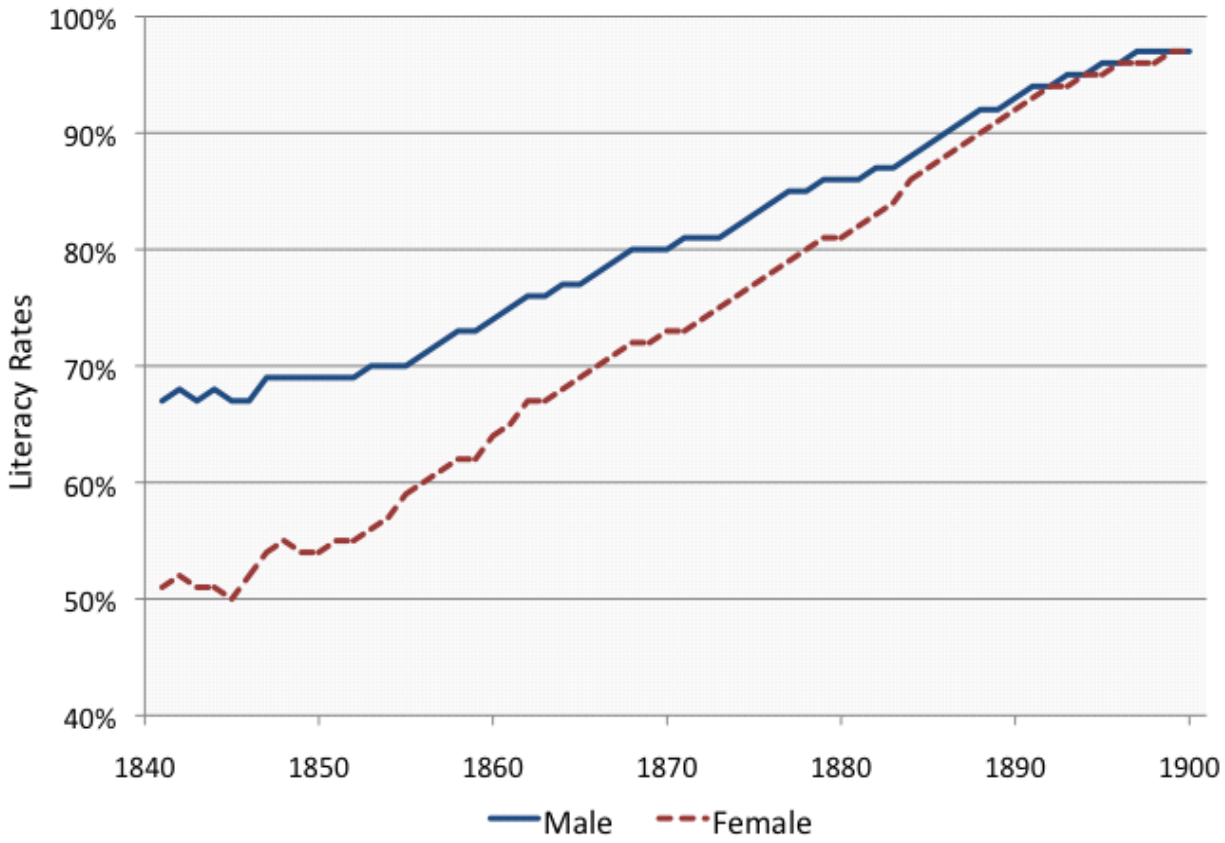
Evolution of the Gender Earninig Ratio - US



Evolution of the Gender Literacy Gap - England



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  - $w^F \tau \equiv$ opportunity cost of raising a child

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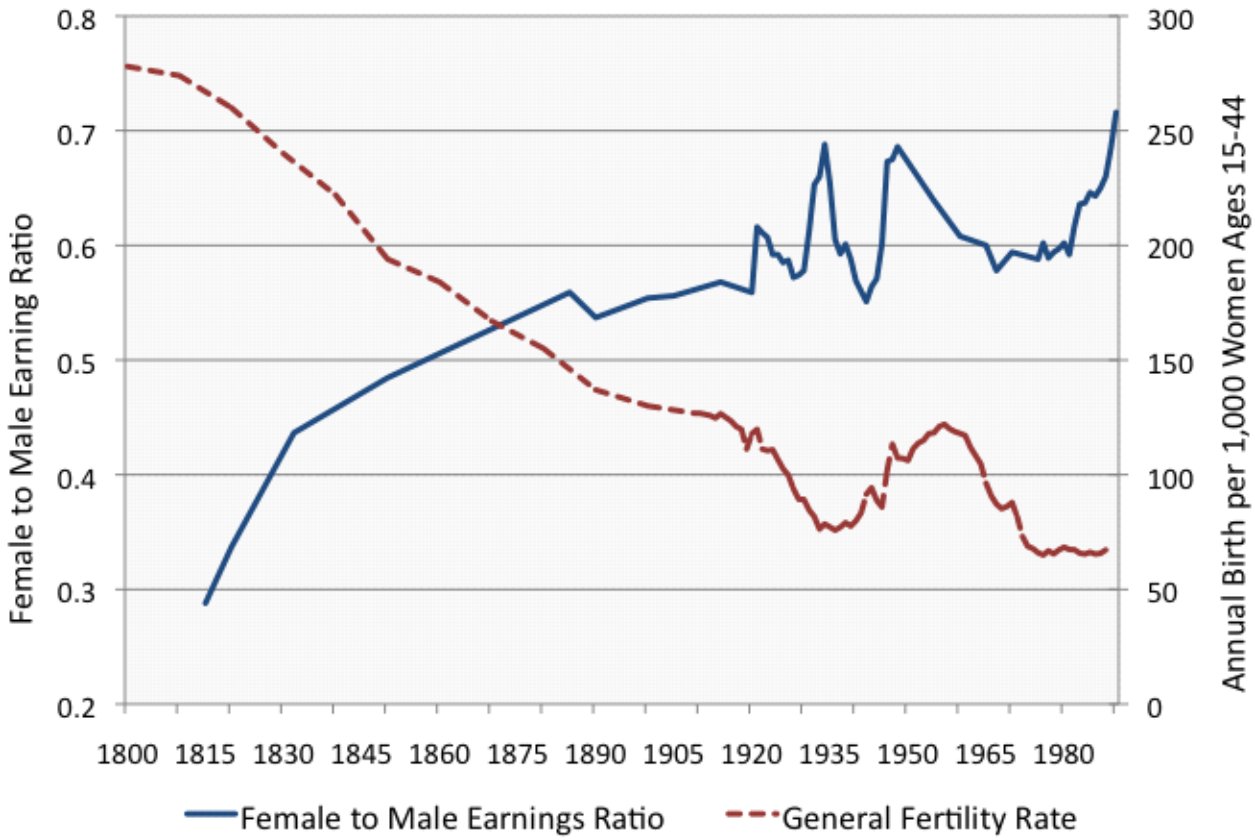
Women's Relative Wages and Fertility - US



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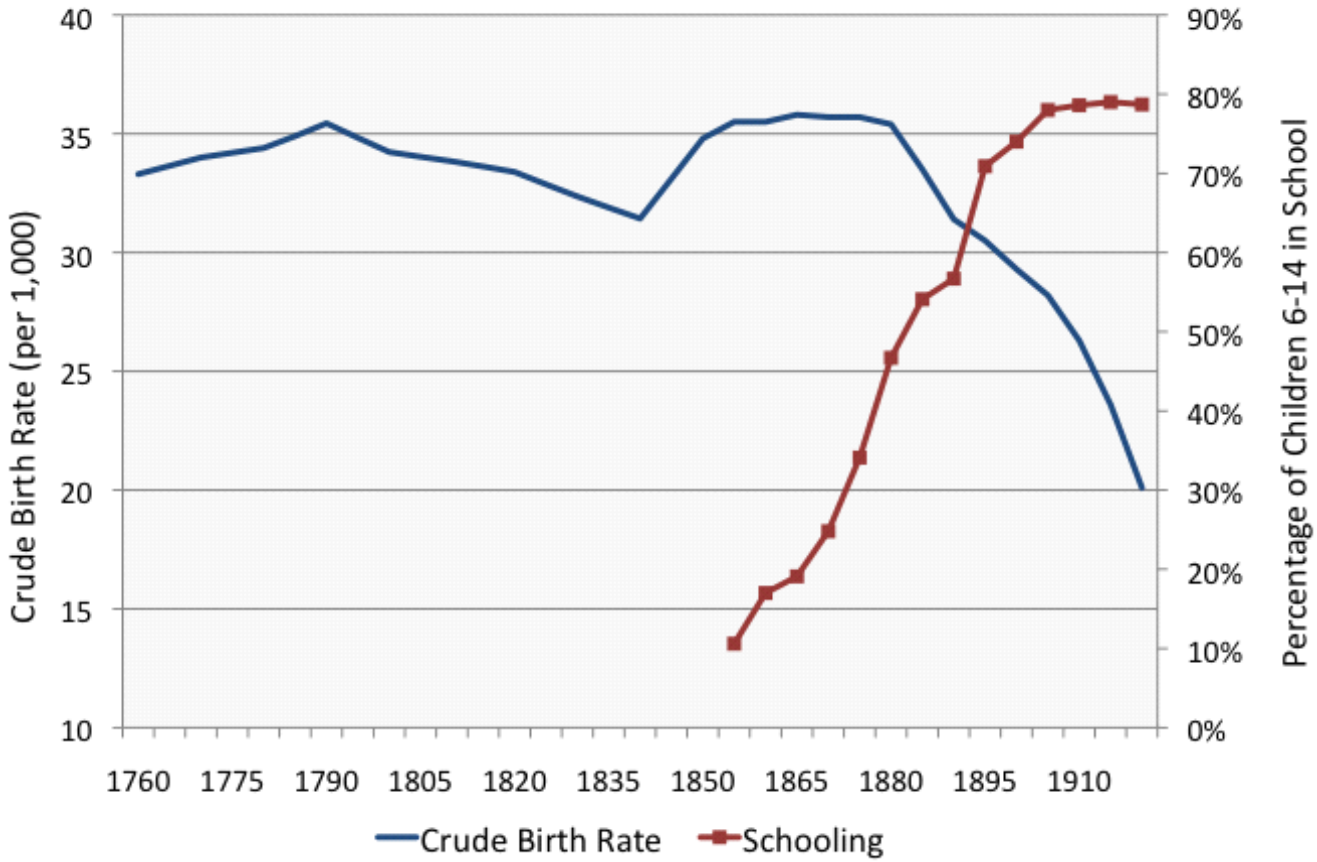
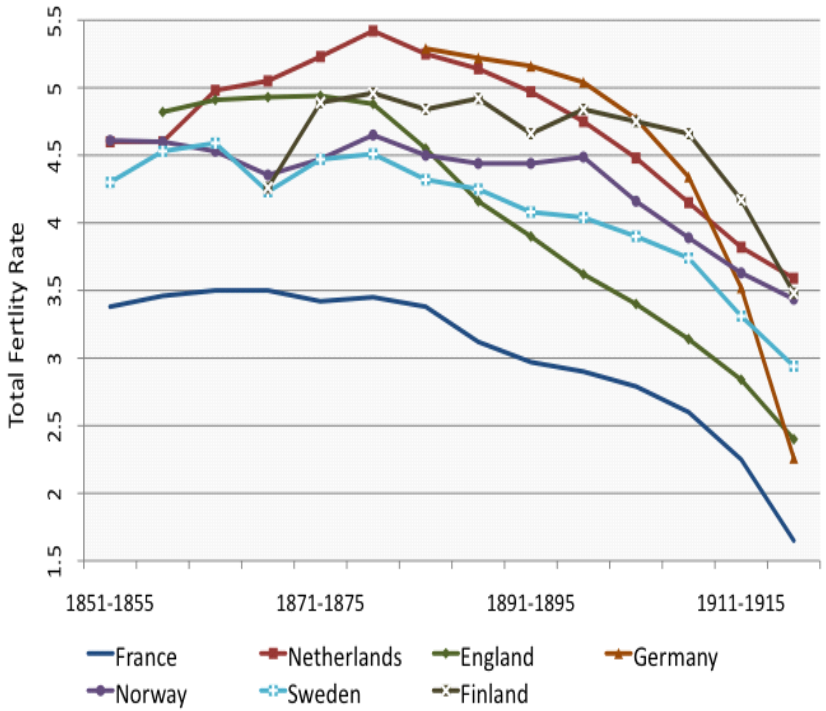
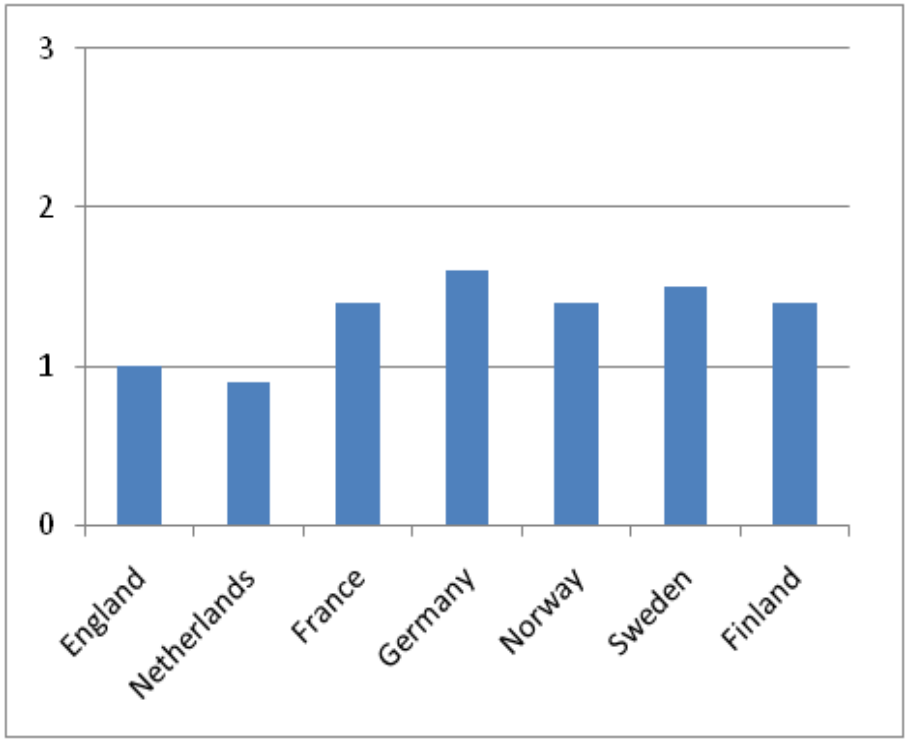
Human Capital Fomration and the Fertility Decline - England



Growth Rates 1870-1913 and DT





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  - Changes in the civil service examination system and their impact on the return to human capital
    - Adverse effect on family size of the likelihood of taking the exam (Shiue, JOEG 2017)
- Ireland (1911)
  - Adverse effect of education attainment on fertility rates (Fernihough, JOEG 2017)

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$$\frac{\gamma\beta h_e(e, g)}{h(e, g)} = \frac{(1 - \gamma) y n \tau^e}{y[1 - n(\tau^q + \tau^e e)]}$$

Optimization

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$$\frac{\gamma\beta h_e(e, g)}{h(e, g)} = n\tau^e \quad \Longrightarrow \quad \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)}$$

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$$\beta h_e(e, g)(\tau^q + \tau^e e) = \tau^e h(e, g)$$

Optimization

$$n = \gamma / (\tau^q + \tau^e e)$$

$$\tau^e h(e, g) = \beta h_e(e, g)(\tau^q + \tau^e e)$$

Optimization

$$n = \gamma / \left(\tau^q + \tau^e e\right)$$

$$\tau^e h(e, g) = \beta h_e(e, g)\left(\tau^q + \tau^e e\right)$$

$\Longrightarrow$

$$e = e(g, \beta, \tau^e, \tau^q),$$

$$n = \gamma / \left[\tau^q + \tau^e e(g, \beta, \tau^e, \tau^q)\right]$$